UNIT 2C
Projectile Motion
**Projectile Motion**

Projectile Motion is similar to 1-D Kinematic Motion.  
Difference: Projectile Motion is 2-D (x and y motion)  
Kinematic Motion is 1-D. (x or y motion)

The Kinematic equations we learned already, as shown below still apply.

\[
V = \frac{d}{t}, \quad A = \frac{\Delta V}{t}
\]

\[
d_x = V_{ix} t + \frac{1}{2} at^2
\]

\[
d_y = V_{iy} t + \frac{1}{2} gt^2
\]

These equations come from

\[
d = V_i t + \frac{1}{2} at^2
\]

“They are components”

\[
V_{fx} = V_{ix} + at
\]

\[
V_{fy} = V_{iy} + gt
\]

\[
V_{fx}^2 = V_{ix}^2 + 2ad
\]

\[
V_{fy}^2 = V_{iy}^2 + 2gd
\]

**General Sketch of Projectile Path**

“Parabolic Path” sq. term \( y = x^2 \)

\( V_{i} = 20 \text{ m/s} \)

\( V_{ytop} = 0 \text{ m/s} \), \( V_x \) is always constant

Time up \( = \) time down

Range to midpoint \( = \) Range from midpoint to end point

Launch angle from the horizontal \( = \Theta \) (theta)
- \( V_x \) always the same (no horizontal acc.) – look at the size of the vectors above
- \( V_y \) decreases going up and then increases going down – look at vectors above
- Points of symmetry should have the same velocities, just opposite direction for \( V_y \)’s
- Acc. Is \( g = 9.81 \text{ m/s}^2 \) for y-dir
- Ignore air friction unless otherwise stated.

\[
d_y = V_{i,y} t + \frac{1}{2} g t^2
\]

\[
+ \frac{1}{2} (-9.81)
\]

Dir.

Sign conv.

Typically, unless otherwise specifically stated, there is no acceleration in the x-direction. All x-direction when rolls off cliff at t=0 seconds!

**Projectile Cases**

“cliff” \( V_i = 10 \text{ m/s} \)

**Knowns**

- \( V_i = 10 \text{ m/s} \)
- \( V_{iy} = 0 \text{ m/s} \)
- \( dy(h) = 30 \text{ m} \)
- \( g = 9.8 \text{ m/s}^2 \)

**Knowns**

- \( V_i = 60 \text{ m/s} \)
- \( V_{iy} = ? \)
- \( T = 5 \text{ sec.} \)
- \( V_{ix} = ? \)

**a)** How long does it take the ball to hit the ground?

- \( V_{iy} = 0 \) so \( V_{iy} t \) goes to 0
- \( dy = V_{i,y} t + \frac{1}{2} g t^2 \)

**b)** Horizontal distance (aka range)

- no acceleration in x-direction
- \( dx = V_{i,x} t + \frac{1}{2} a t \)
- use time from a) above

**EX**

\( \Theta = 30^\circ \)

**Knowns**

- \( V_i = 60 \text{ m/s} \)
- \( V_{iy} = ? \)
- \( T = 5 \text{ sec.} \)
- \( V_{ix} = ? \)
Now we have to resolve $V_i$ into $x$ and $y$ components to get $V_{iy}$ and $V_{ix}$.

![Vector diagram]

SOHCAHTOA

$\sin \theta = \frac{\text{opp.}}{\text{hyp.}} \Rightarrow V_{iy} = V_{ix}/60\text{ m/s}$ or $\cos 30 = V_{ix}/60\text{ m/s}$

$V_{ix} = 51.6\text{ m/s}$

$V = \sqrt{\sin 30 = V_{iy}/60\text{ m/s}}$ or $\cos 60 = V_{iy}/60\text{ m/s}$

$V_{iy} = 30\text{ m/s}$

A) Maximum height of projectile?

$dy = V_{iy}t + \frac{1}{2}gt^2$

$(30 \text{ m/s})(55) + \frac{1}{2}(-9.8\text{ m/s})(55)^2 = 150\text{m} - 122.5 = 27.5\text{m}$

Total Distance in $y = 57.5\text{m}$

**Ball Launched Horizontally off Cliff Example:**

I throw a ball off the edge of a 15.0m tall cliff. If I threw it horizontally at 8.0m/s…

a) How much time did it take to fall?

b) How far from the base of the cliff does it hit the ground?

c) How fast is it moving vertically when it hits the ground?

d) What is its total velocity when it hits the ground?

a) THINK VERTICAL

We’re talking about something falling, and that is vertical motion, so we will only use vertical ideas and numbers. It actually would take the exact same amount of time for the object to hit the ground if I just dropped it straight down from the edge of the cliff (diagram 7-1 on page 134), so let’s just calculate the time to fall that way. Remember to think vertically…

$d = v_i t + \frac{1}{2}at^2 \quad \Leftarrow \text{initial velocity in the y-direction is zero}$

$d = \frac{1}{2}at^2$

$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(-15.0\text{m})}{-9.81\text{m/s}^2}} \quad \Leftarrow \text{Moving down the cliff is a negative displacement & acceleration}$

$t = 1.75\text{ s}$
b) THINK HORIZONTAL
Well, we know it was in the air for 1.75s (from the previous question), and it was moving at a constant speed of 8.0m/s in the x-direction the whole time, so...

\[ v = \frac{d}{t} \]
\[ d = vt \]
\[ = (8.0\text{m/s})(1.75\text{s}) \]
\[ d = 14\text{m} \]

It will move 14m horizontally, so it hits the ground 14m away from the base of the cliff.

c) THINK VERTICAL
It has been accelerating down the whole time. We know that gravity is causing this acceleration, and that it wasn’t moving vertically initially, so we can figure out how fast it is going (vertically) when it hits the ground.

\[ v_f^2 = v_i^2 + 2ad \]
\[ = (0)^2 + 2(9.81\text{m/s}^2)(15.0\text{m}) \]
\[ v_f = 17\text{ m/s} \]

d) It’s total velocity is found by adding the horizontal and vertical components of the velocity to find the resultant.

\[ c^2 = a^2 + b^2 \]
\[ = (8.0\text{m/s})^2 + (17\text{m/s})^2 \]
\[ c = 19\text{m/s} \]

\[ \tan \theta = \frac{\text{opp}}{\text{adj}} \]
\[ = \frac{(8.0\text{m/s})}{(17\text{m/s})} \]
\[ \theta = 25^\circ \]

Although there will always be slight differences in actual problems, this is the standard sort of questions that you will be asked for these types of questions.

**Ball Launched at an Angle Example:**

You kick a soccer ball at an angle of 40\(^\circ\) above the ground with a velocity of 20m/s.

a) How high will it go?

b) How much time does it spend in the air?

c) How far away from you will it hit the ground (range)?

d) What is the ball’s velocity when it hits the ground?
Before we can calculate anything else, we first need to break the original velocity into components.

- We do this so we have a vertical component to do the first couple calculations with. The horizontal component will be used later.
- For now, think vertically!!

\[
\sin \theta = \frac{\text{opp}}{\text{hyp}}
\]

\[
\text{opp} = \sin \theta \times \text{hyp}
\]

\[
\text{opp} = \sin 40^\circ \times (20 \text{ m/s})
\]

\[
\text{opp} = v_y = 13 \text{ m/s}
\]

\[a)
\]

\[
v_f^2 = v_i^2 + 2ad
\]

\[d = \frac{v_f^2 - v_i^2}{2a} = \frac{0^2 - (13 \text{ m/s})^2}{2(-9.81 \text{ m/s}^2)} = 8.6 \text{ m}
\]

\[b)
\]

\[a = \frac{v_f - v_i}{t}
\]

\[t = \frac{v_f - v_i}{a} = \frac{0 - 13 \text{ m/s}}{-9.81 \text{ m/s}^2} = 1.3 \text{ s} \leftarrow \text{But this is only the time to the halfway point}
\]

Final answer \( t = 2.7 \text{ s} \)

c) It is moving at a constant velocity horizontally the whole time I just figured out, so let’s use the diagram that was drawn above to get the horizontal component of the velocity.

\[
\cos \theta = \frac{\text{adj}}{\text{hyp}}
\]

\[
\text{adj} = \cos \theta \times \text{hyp}
\]

\[
\text{adj} = \cos 40^\circ \times (20 \text{ m/s})
\]

\[
\text{adj} = v_x = 15 \text{ m/s}
\]

\[v = \frac{d}{t}
\]

\[d = vt
\]

\[
= 15 \text{ m/s} \times (2.7 \text{ s})
\]

\[d = 41 \text{ m} \leftarrow \text{This is the horizontal distance (range) that it has traveled.}
\]

d) The ball’s velocity when it hits the ground is exactly the same as when it was originally launched… 20 m/s at 40° up from the horizontal.
Objective: Use your Review book, Reference Table & Notepacket to complete the notesheet.

1. “The motion of an object traveling in a two-dimensional plane can be described by separating its motion into the ____________ and ____________ components of its ____________ , ____________ , and ______________ .

A component parallel to the horizon is a ____________________ and a component at right angles to the horizon is a ____________________ .

If air resistance is neglected, an example of two-dimensional motion is the motion of a cannonball projected (launched or fired) near the surface of the Earth at an angle to the horizontal. If gravity is the only unbalanced force acting on the cannonball, the ____________ of the ball’s motion is identical to that of a ______________ , and the ______________ is ______________ . Although the two motions occur simultaneously, the two components of the motion are ______________ . Thus, if the object’s initial velocity is known, the motion of the object in Earth’s gravitational field can be described by the ____________ of the two motions.

2. Vocabulary:

Projectile: ________________________________

Range: ________________________________

Muzzle velocity: ________________________________

Trajectory: ________________________________

Horizontal component: ________________________________

Vertical component: ________________________________

(OVER)
Notesheet: Two-Dimensionals Motion and Trajectories

A Projectile Fired at an Angle

Method to solve problems:

a) Decompose the initial velocity vector

b) Add $v_{iy}$ and $v_{ix}$ to form $v_i$

c) SOHCAHTOA review

Horizontal component: $v_{ix} = \text{________}$

Vertical component: $v_{iy} = \text{________}$

d) Divide your solution space into two columns: x – horizontal, y – vertical

<table>
<thead>
<tr>
<th>horizontal</th>
<th>vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>equations used:</td>
<td>equations used:</td>
</tr>
</tbody>
</table>

e) Show how the horizontal and vertical velocities change with respect to time:

f) Sketch 5 sets of x and y velocity vectors on the trajectory below:
**Example:**
A projectile is launched from the ground (origin 0,0) at an initial velocity of \( V = 120 \text{ m/s} \), at an angle of \( 60^\circ \).

1. Calculate \( V_{ix} \)

2. Calculate \( V_{iy} \)

3. Complete the table below for the entire time of flight

<table>
<thead>
<tr>
<th>Time</th>
<th>( V_x )</th>
<th>( V_y )</th>
<th>horizontal disp.</th>
<th>vertical disp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.5 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.6 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.5 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.5 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21.2 secs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Name____________________________________ Date_______
Regents Physics Mr. Morgante
Learning activities: Two- Dimensional Motion and Trajectories
Objective: Review projectile motion/ trajectories using simple demos

Activity #1  Rulers & Pennies
2. Sketch probable paths of both pennies w/ coordinate system.  
   *Show direction of “g”, Show velocity vectors at instant of launch, halfway to floor, the instant pennies strike floor.*
3. Do both pennies appear to hit the floor at the same time?________
4. What should be the result of this experiment if the horizontal launch velocity of the penny was greatly increased?  
   _______________________________

Activity #2  Ball bearing projectile machine
2. Sketch probable paths of both balls w/ coordinate system.  
   *Show direction of “g”, Show velocity vectors at instant of launch, halfway to floor, the instant balls strike floor.*
3. Do both balls appear to hit the floor at the same time?________
4. What should be the result of this experiment if the horizontal launch velocity of the ball was greatly increased?  
   _______________________________

5. Use a ball and a penny and conduct the ruler/penny experiment with the ball balanced on the ruler. Describe your results:  
   (OVER)
Activity #3  “Cliff case” &  “Cannon case” diagram in space below
1. Posters must be done in color.
2. Each case must have both horizontal and vertical velocity components drawn on trajectory path.
3. Cliff and cannon case must show direction of “g”.
4. Cannon case must include Range, trajectory, projectile vocab.
5. Cannon case must show method to find x and y components of launch velocity using SOHCAHTOA and Pythagorean Theorem

Activity #4  Trajectory rods
1. Use the plastic trajectory rods to model both the “cannon” and “cliff” cases.
2. Give rough sketches below for the following cases:
   Cliff(velocities at launch)     Cannon (velocities at launch)
   Large horizontal velocity      Large horizontal velocity and small vertical velocity
   Small horizontal velocity      Small horizontal velocity and large vertical velocity
   No horizontal velocity         Horizontal and vertical velocities are equal (angle?)
A projectile is launched from the ground (origin 0,0) and remains in the air for **12 seconds** before reaching ground level again. If the **range** = **3,000 m,**

1. Show your coordinate system and sketch the approximate trajectory below:

![Diagram](image)

2. Calculate \( v_{ix} \):

   \[ 2. \text{______________} \]

3. Calculate \( v_{iy} \):

   \[ 3. \text{______________} \]

4. Calculate the maximum height reached by the projectile:

   \[ 4. \text{______________} \]

5. Calculate the launch angle from horizontal

   \[ 5. \text{______________} \]

6. **On your trajectory sketch above,**
   - draw the \( x \) & \( y \) velocity vectors at \((0,0)\), \((3000 \text{ m},0)\), and the midpoint of the trajectory.
Regents Physics

Projectile Motion Worksheet #3

Analyze each question, underline or circle essential data, sketch the question, √ check your answers.

1. A vector makes an angle, \( \theta \), with the horizontal.
   The horizontal and vertical components of the vector will be equal in magnitude if angle \( \theta \) is
   (1) 45°  (2) 60°  (3) 75°  (4) 30°

2. A basketball player jumped straight up to grab a rebound.
   If she was in the air for 0.8 second, how high did she jump?
   (1) 0.50 m  (2) 0.78 m  (3) 3.1 m  (4) 1.2 m

3. A ball is thrown at an angle of 38° to the horizontal. What happens to the magnitude of the ball’s vertical acceleration during the total time interval the ball is in the air?
   (1) It decreases, then increases  (2) It increases, then decreases
   (3) It decreases, then remains the same  (4) It remains the same

4. A ball thrown vertically upward reaches a maximum height of 30. meters above the surface of the Earth. At its maximum height, the speed of the ball is
   (1) 9.8 m/s  (2) 24 m/s  (3) 3.1 m/s  (4) 0.0 m/s

5. A 0.2 kilogram red ball is thrown horizontally at a speed of 4 meters per sec. from a height of 3 meters. A 0.4 kilogram green ball is thrown horizontally from the same height at a speed of 8 meters per second. Compared to the time it takes the red ball to reach the ground, the time it takes the green ball to reach the ground is
   (1) one – half as great  (2) twice as great  (3) the same  (4) four time as great

(OVER)
6. A projectile is fired at a launch velocity of 113.2 m/s at an angle of 60° to the horizontal. How long will it take the projectile to reach the highest point in the trajectory?
(1) 5.0 s  (2) 10. s  (3) 20. s  (4) 100. s

7. The diagram below represents the path of an object after it was thrown. What happens to the object’s acceleration as it travels from A to B? (Neglect air resistance, friction)

(1) it decreases  (2) it increases  (3) remains the same

8. A student throws a stone upward at an angle of 45° to the horizontal. Which statement best describes the stone at the highest point that it reaches?
(1) Its acceleration is zero.
(2) Its acceleration is at a minimum.
(3) Its horizontal and vertical velocity vectors are equal.
(4) Its horizontal velocity vector is equal.

9. A baseball player throws a ball horizontally. Which statement best describes the ball's motion after it is thrown? (Neglect air resistance)
(1) Its vertical speed remains the same and its horizontal speed increases.
(2) Its vertical speed remains the same and its horizontal speed remains the same.
(3) Its vertical speed increases and its horizontal speed increases.
(4) Its vertical speed increases and its horizontal speed remains the same.

Bonus  Bonus  Bonus  Bonus  Bonus  Bonus  Bonus  Bonus  Bonus  Bonus

10. An object is allowed to fall freely from rest near the surface of a planet. If the object falls 54 meters in the first 3.0 seconds after it is released, What is the acceleration due to gravity on this planet?
(1) 6.0 m/s²  (2) 9.8 m/s²  (3) 12 m/s²  (4) 18 m/s²
A Projectile Fired Horizontally

A soccer ball is kicked horizontally from the edge of a high cliff at $V_{\text{horizontal}} = +10 \text{ m/s}$. 

a. Complete the sketch below; show coordinate system:

```
  o
```

b. Calculate and complete the data table below. **Wind resistance is negligible.**

<table>
<thead>
<tr>
<th>Time</th>
<th>Horizontal displacement</th>
<th>Horizontal velocity</th>
<th>Vertical displacement</th>
<th>Vertical velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 seconds</td>
<td>0 m</td>
<td>+10 m/s</td>
<td>0 m</td>
<td>0 m/s</td>
</tr>
<tr>
<td>1 second</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 seconds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 seconds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 seconds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Show Sample Calculations Below:
1. A skateboarder rolls 25.0 m down a hill that descends at an angle of 20.0° with the horizontal. Find the horizontal and vertical components of the skateboarder's displacement.

2. A stone is thrown at an angle of 30.0° above the horizontal from the top edge of a cliff with an initial speed of 12 m/s. A stopwatch measures the stone’s trajectory time from the top of the cliff to the bottom at 5.6 s. What is the height of the cliff? (Disregard air resistance)

3. A track star in the long jump goes into the jump at 12 m/s and launches herself at 20.0° above the horizontal. How long is she in the air before returning to Earth?

4. A model rocket flies horizontally off the edge of the cliff at a velocity of 50.0 m/s. If the canyon below is 100.0 m deep, how far from the edge of the cliff does the model rocket land?

5. A firefighter 50.0 m away from a burning building directs a stream of water from a fire hose at an angle of 30.0° above the horizontal. If the velocity of the stream is 40.0 m/s, at what height will the stream of water strike the building?
61 A baseball player throws a baseball at a speed of 40. meters per second at an angle of 30.° to the ground. The horizontal component of the baseball’s speed is approximately

(1) 15 m/s  
(2) 20. m/s  
(3) 30. m/s  
(4) 35 m/s

62 Projectiles are fired from different angles with the same initial speed of 14 meters per second. The graph below shows the range of the projectiles as a function of the original angle of inclination to the ground, neglecting air resistance.

The graph shows that the range of the projectiles is

1. the same for all angles
2. the same for angles of 20.° and 80.°
3. greatest for an angle of 45°
4. greatest for an angle of 90.°
64 Four different balls are thrown horizontally off the top of four cliffs. In which diagram does the ball have the shortest time of flight?

(1) $m = 1.0 \, \text{kg}$  
$m = 50. \, \text{m/s}$  
$s = 125 \, \text{m}$  

(2) $m = 0.25 \, \text{kg}$  
$m = 35. \, \text{m/s}$  
$s = 375 \, \text{m}$  

(3) $m = 0.50 \, \text{kg}$  
$m = 40. \, \text{m/s}$  
$s = 250 \, \text{m}$  

(4) $m = 0.10 \, \text{kg}$  
$m = 20. \, \text{m/s}$  
$s = 450 \, \text{m}$

59 A 2-kilogram block is dropped from the roof of a tall building at the same time a 6-kilogram ball is thrown horizontally from the same height. Which statement best describes the motion of the block and the motion of the ball? [Neglect air resistance.]

1. The 2-kg block hits the ground first because it has no horizontal velocity.
2. The 6-kg ball hits the ground first because it has more mass.
3. The 6-kg ball hits the ground first because it is round.
4. The block and the ball hit the ground at the same time because they have the same vertical acceleration.

62 The path of a projectile fired at a $30^\circ$ angle to the horizontal is best described as

1. parabolic
2. linear
3. circular
4. hyperbolic
61. An artillery shell is fired at an angle to the horizontal. Its initial velocity has a vertical component of 150 meters per second and a horizontal component of 280 meters per second. What is the magnitude of the initial velocity of the shell?

(1) $9.0 \times 10^4$ m/s  
(2) $4.1 \times 10^2$ m/s  
(3) $3.0 \times 10^2$ m/s  
(4) $1.1 \times 10^2$ m/s

62. The diagram below shows a projectile moving with speed $v$ at the top of its trajectory.

![Diagram of projectile motion]

Which vector best represents the acceleration of the projectile in the position shown?

(1)  
(2)  
(3)  
(4)
In the diagram below, a stationary observer on the ground watches as a seagull flying horizontally to the right drops a clamshell.

Which diagram best represents the path of the falling clamshell as seen by the observer? [Neglect air resistance.]
Base your answers to questions 56 through 58 on the diagram and information below.

A machine launches a tennis ball at an angle of 45° with the horizontal, as shown. The ball has an initial vertical velocity of 9.0 meters per second and an initial horizontal velocity of 9.0 meters per second. The ball reaches its maximum height 0.92 second after its launch. [Neglect air resistance and assume the ball lands at the same height above the ground from which it was launched.]

**56** The speed of the tennis ball as it leaves the launcher is approximately

| (1) 4.5 m/s | (2) 8.5 m/s | (3) 13 m/s | (4) 18 m/s |

**57** The total horizontal distance traveled by the tennis ball during the entire time it is in the air is approximately

| (1) 23 m | (2) 17 m | (3) 8.3 m | (4) 4.1 m |

**58** The diagram below shows the muzzle of a cannon located 50 meters above the ground. When the cannon is fired, a ball leaves the muzzle with an initial horizontal speed of 250 meters per second. [Neglect air resistance.]

Which action would most likely increase the time of flight of a ball fired by the cannon?

1. pointing the muzzle of the cannon toward the ground
2. moving the cannon closer to the edge of the cliff
3. positioning the cannon higher above the ground
4. giving the ball a greater initial horizontal velocity

Note that question 58 has only three choices.
Projectile Motion Free Response (SHOW ALL WORK)

1. A rock is thrown at 25 ft/sec, 45° above the horizontal. How long will it be in the air?

2. Bullet #1 is fired horizontally at 900 m/sec while Bullet #2 is dropped vertically from the same height. Which bullet will land first? Explain your answer in a sentence or two below (we neglect air resistance, of course). What should the angle be to get the maximum height possible with Bullet #1? Can you show a mathematical proof below for extra credit (make up some numbers if you would like)?

3. A physics student, frustrated with the class, releases his tensions by throwing stones into Beaver Dam Lake later on that day. One particular stone is fired at 55° from horizontal with a vertical speed of 30 m/sec. a) How high does the stone go? b) How far across the lake does the stone go?
During the 16th century, Galileo Galelei did a series of experiments along with some good, sound reasoning. He concluded that all objects, no matter what their weight, would fall at the same rate if friction with the air could be eliminated.

His reasoning was this: Two identical spheres are dropped simultaneously. Everyone will agree they must fall at the same rate. We bring the spheres closer together on successive drops until they are touching as they fall. Again, everyone will agree they still fall at the same rate. Finally, we weld the two together and they will fall at the same rate they did before they were joined. Thus the welded object, with twice the weight of a single sphere will fall no faster.

Although Galileo had crude timing devices and could not accurately measure the acceleration of a free-falling object, he could measure the rate at which spheres rolled down inclined planes. Mathematically extrapolating to a 90° incline, he calculated the free-fall acceleration without actually measuring it directly. Although we take this approach to science as routine, in Galileo’s time it was a leap toward abstract generalization that was truly revolutionary.

The acceleration of a free-falling object is $9.81 \text{ m/sec}^2$. When we use coordinates which are positive in the up direction, we will take the acceleration of gravity to be $-9.81 \text{ m/sec}^2$.

*And now some problems:*

1. How far will a rock fall in 6 seconds if it is released from rest?

2. A ball is thrown down at a initial velocity of 18 m/sec. How far will it fall in the first 4 seconds?

3. A ball is thrown up at 22 m/sec. How far will it be above the ground in (a) 2 seconds? (b) 4 seconds?

4. Freddy Frog takes a swan diver from a bridge that is 53 meters above the water. Freddy is an expert and can streamline himself to eliminate wind resistance. How long will it take him to reach his aquatic homeland?

5. Often it is necessary to work in units of cm or ft, so you need to know gravity in these units.
   
   a) Convert $9.8 \text{ m/sec}^2$ to cm/sec$^2$.
   b) Convert $9.8 \text{ m/sec}^2$ to ft/sec$^2$.

6. How far will a rock fall (in feet) if it drops for 8.2 seconds?
7. How long does it take a needle to drop 2.6 cm?

There is no #8!!!!

As we saw in the previous chapter, when two dimensions are involved we may treat each dimension individually and independently. Thus a falling object will accelerate at \(-9.8 \text{ m/sec}^2\) whether it is released from rest or thrown horizontally. In both cases the initial vertical velocity is zero, so at each moment after release, they will have fallen an identical vertical distance.

9. A rock is thrown horizontally at 18 m/sec. Where is it 1.5 seconds later?

10. A rock is thrown horizontally at 5.7 m/sec. Find its velocity and the direction of its motion 3.6 seconds after release.

11. A can of Skoal is kicked horizontally at 4.6 m/sec off a cliff 12.4 m high.
   a) How long will it take to land?
   b) How far from the cliff will it land?

12. A man fired a bullet horizontally from 1.8 m above the ground. If the bullet’s speed is 415 m/sec, how far away will it land?

13. A marble, rolling off the edge of a table is observed to hit the floor 0.77 m from the table. If the table top is 0.86 m high, how fast was the marble traveling when it left the table?

14. A colony of troglodytes has been in a lengthy feud with its neighbors on the adjacent cliff. Colony A finally develops an important military breakthrough; it rolls bombs off its cliff at known rates of speed, thus gaining pinpoint accuracy in its attacks. If the cliffs are separated by 42 m and a bomb is rolled at 6.0 m/sec, how far down the cliff will it land?

15. The troglodyte war continues and a particularly offensive member of colony B is located 110 m below the top. At what speed must a bomb be rolled to get him?

6. Freddy Frog returns for another diving competition. This time from the 13 m board, he takes off at 6.5 m/sec.
If his initial velocity is horizontal, at what angle and at what total speed does Freddy hit the water?

17. Ford tests a new safety system by driving a car off a high cliff. If the vehicle leaves the edge with a speed of 68 ft/sec, what will be its speed and direction in 1.5 seconds?

18. A family of trolls communicates with another family at the bottom of a two-tiered cliff as shown. The trolls tie messages to rocks and roll them off the top cliff at the minimum speed necessary to just miss the 2nd cliff and land below.
   a) What is the velocity chosen?
   b) How far from the bottom cliff will the projectile land?

19. This time the gregarious troll family wishes to fire off a message to their neighbors who live under the overhang of a cliff. With what velocity must they roll the rock, and how far under the overhang will it hit?

20. Find the general equation for problem 18, using the parameters shown at right.

*So far we have considered projectile problems where the initial velocity has always been horizontal. We have been simplifying the mathematics while installing the concept that two independent motions are taking place simultaneously. This same concept will apply to motion, which begins at some angle other than horizontal. Before studying general projectile motion, however, we must review some of our previous work.

21. A rock is dropped from a 42 ft high cliff. How long does it take to touch the ground?

22. A bolt drops out of a bridge. How fast is it traveling (in m/sec) 2.6 seconds after its release?

23. A soccer ball is kicked vertically up at 22 m/sec. How long will it take to reach the peak of its flight?

24. A gun fires a bullet vertically up at 1330 ft/sec. How long will it take to reach its highest point? (neglect friction, of course)

25. A rock thrown up at 8.6 m/sec will take how long to hit the ground?
26. A baseball thrown vertically up is aloft 4.8 seconds before returning to earth. What was its initial velocity in ft/sec?

27. A raindrop, mysteriously falling on the moon, hits at 57.4 m/sec. The moon’s gravity is 1.6 m/sec², so how long was the drop falling?

28. If a rock is thrown vertically up at 17.3 m/sec, how long will it take before it is falling at –8.8 m/sec?

*Now for some good stuff.

29. A Brazilian aborigine fires a dart from his blowgun at 41 m/sec, aimed 32° above the horizontal.
   a) What is its initial vertical velocity?
   b) How long does it take to return to earth?

30. Jimmy Jones steps up to bat for a whopper of a hit. The baseball leaves the bat at 76 m/sec, 57° above the horizontal. How long is it in the air?

31. A rock is thrown at 15.2 ft/sec, 41° above the horizontal. How long will it be in the air?

32. A Roman centurian fires off a vat of burning pitch from his catapult at 18.2 m/sec, 37° above the horizontal.
   a) How long is it in the air?
   b) What is the horizontal component of velocity?
   c) How fat does it get before landing?

33. A bullet is fired at 677 m/sec, 35° above the horizontal. How far does it get before landing? (we neglect air resistance, of course)

34. A baseball, thrown at 21.7 m/sec 28° above the horizontal will travel how far before hitting the ground?

35. A rock is thrown so that it is in the air 4.3 seconds and lands 36 m from its starting point. What was the initial vertical and horizontal speed?

36. A slingshot fires a pebble which is in the air for 5.3 seconds and lands 73 m from where it took off. What were the initial vertical and horizontal components of velocity?

*Look over what we are doing in these problems.

First, we consider each as two separate problems: a vertical one and a horizontal one. In case of projectile motion, the vertical velocity is the thing that gets the ball into the air. The greater the vertical velocity, the longer the ball will stay aloft.
The horizontal velocity has no effect whatsoever on how long the ball stays up, but it’s the thing that moves the ball forward. We see, then, that the range of a projectile is a combination of the horizontal speed at which the projectile moves toward its target, and the vertical speed which determines how long the projectile is allowed to keep moving.

37. A B-B is fired at 52° above the horizontal with an initial speed of 36 m/sec. What is its range?

38. A man is shot out of a cannon at 42° with an initial speed of 11.6 m/sec. How far away must he place the safety net?

39. A college freshman, frustrated with finals, releases his tensions by bombarding the adjacent dorm with water balloons. He fires one at 38° from horizontal with a speed of 21.4 m/sec. How far up the building does it get? The dorm is 13.2 m away.

40. Since the great troglodyte war on the previous pages, colony B has rebuilt and has developed its own technology. It can fire projectiles at different angles, though they all are fired at \( v_0 = 26 \text{ m/sec} \). How far from the top of the cliff will a bomb aimed 22° below the horizon land?

41. Where should troglodytes B aim to strike fear into those of colony A who live 88 m below the surface? Remember, the projectile is fired at 26 m/sec.

*You can solve projectile problems in a more general way by assigning variables rather than numerical values to the significant quantities.

42. A rock is kicked off a cliff of height with a velocity \( v \). The acceleration of gravity is \( g \).
   a) How long will it take to land?
   b) How far from the base of the cliff will it land?

43. A disgruntled auto worker pushes a small foreign import off a cliff of height \( h \). If the vehicle lands a distance \( h \) from the base, how fast was it pushed initially?

44. A creative chef cracks walnuts by catapulting them into a wall a distance \( s \) away. If she releases them at velocity \( v \) and angle \( \theta \),
   a) What are the initial horizontal and vertical velocities?
   b) How long is the projectile in the air?
   c) How high does it strike the wall?
45. A disgruntled Japanese fast food operator hurls an Egg McMuffin off a cliff of height \( h \), velocity \( v \) and its initial angle \( \theta \). 
   a) What are its initial \( v_x \) and \( v_y \)?
   b) How long is it in the air? (x and y variables)
   c) How far from the base of the cliff does it land?

46. A projectile is fired vertically up at 35 m/sec. How long will it take the reach the height of 40 m?
   Hint- you need to use the quadratic formula: 
   \[ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

47. A projectile is fired up at 27 m/sec. How long will it be in the air if a bird catches it in mid air, while it’s falling, 25 m from the ground?

48. A ball on a cliff is thrown vertically down at 12 ft/sec. If the cliff is 77 ft high, how long will it take to hit the ground?

49. A call is thrown up at 19 ft/sec off the same 77 ft high cliff. How long will it take to reach the bottom?

50. This time the ball is thrown off the edge of the cliff at 40 ft/sec, 28° above the horizontal. How long will its take to reach the ground? (The cliff is still 77 ft high)

51. Now a ball is fired at 37 m/sec, 41° above the horizontal, up onto a cliff that is 26 m high. How far does it travel horizontally?

52. Finally, a ball is fired at 67° above the horizontal onto a cliff 38 m high, with an initial velocity of 32 m/sec. How far does it move horizontally?

53. A bridge is \( h \) above the round. How long will it take a rock to fall if it’s released from the rest of the bridge? (gravity is \(-g\))
   a) How long will it take if thrown down at velocity \(-v_0\)?
   b) How long will it take if thrown up at \( v_0 \)?

54. A rock is thrown at velocity \( v_0 \) at angle \( \theta \) on level ground.
   a) What is its initial vertical velocity?
   b) How long will it remain in the air?
   c) How far will it get before hitting the ground?
A football is kicked at an angle of 37° with a velocity of 20 m/s. The kick is attempted 36 meters from goalposts, whose crossbar is 3 meters above the ground. If the football is directed correctly between the goalposts, will it pass over the bar and be a field goal? Show why or why not.